Propagation of Experimental Error in Calculations

We will be making a lot of measurements in subsequent labs. Each of these will have an error associated it. Our problem is that we will be performing calculations with these measurements and then comparing the answers of various calculations to each other and to accepted values. In order to do this, we need to know the error in our answers.

There are many ways to address this issue.

a) Tracking significant digits is a very simple way to estimate the error in a calculated number, but it yields only a crude estimate of the real error.

b) Propagation of errors is a less crude method, but yields a better estimate of the true error. There are better, but more complex statistical techniques, but for the near future, propagation of error is the method that we will use.

General Example

We make two measurements: A ±a and B ±b where ±a and ±b are the range of the errors in A and B respectively.

Definitions: absolute error is the range of values for a given measurement.
relative error is the range of values for a measurement as a proportion of the measurement.

for measurement A if ±a is the absolute error in A
then ±(a/A) is the relative error in A

C = A + B what is the value of ±c (the error range in C)?
if C = A + B then ±c = ±(a+b)
if C = A - B then ±c = ±(a+b)
The error in a sum or difference is the sum of the absolute errors.

D = A x B what is the value of ±d (the error range in D)?
if D = A x B then ±(d/D) = ±((a/A) + (b/B)) ±d = ±((a/A) + (b/B)) x D
if D = A / B then ±(d/D) = ±((a/A) + (b/B)) ±d = ±((a/A) + (b/B)) x D
The relative error of a product or quotient is the sum or the relative errors.

Examples

A= 3 ± 0.3
B = 5 ± 0.1

A + B = 3 + 5 = 8 ±(0.3 + 0.1) = 8 ±0.4
The absolute errors are added together.
The same procedure would have been followed if we were subtracting.

A x B = 3 x 5 = 15 ±(((0.3/3) + (0.1/5)) x 15) = 15 ±((0.1 + 0.02) x 15) = 15 ±(0.12 x 15) = 15 ±1.8
First, we calculated a relative error by dividing the absolute errors by the measurements themselves.
Second, we add the relative errors together.
Last, we multiply the total relative error by the answer to calculate the absolute error in the answer.
The same process would have been followed for a division.